The intensification of the sublimational dehydration process by the use of sorbents can be evaluated quantitatively from the variation in time in the moisture content of the material–drying curves in Fig. 4.

The time taken to dry a moist brick to a given moisture content (2%) is 220 min (modification I). When the sorbent is placed on the surface to be sublimated (modification II) the duration of the dehydration is cut by 18% (τ=180 min). The dehydration time is affected significantly by the organization of the layer structure: modification III τ=130 min and modification IV 90 min, i.e., the duration of drying is reduced by 40% and 60%, respectively.

Thus, the duration of dehydration in a vacuum is shortened considerably by the use of sorbents in direct contact with the material and by the correct organization of the sorbent–material layer structure.

This kind of contact mass exchange takes on especial significance for the low-temperature drying of highly thermolabile materials. In particular, for a number of products of biological origin a 10-15° increase in temperature causes the sudden inactivation of ferments and denaturation of cellular albumin. In this case the cost of sorbents and their requirement for periodical regeneration is of secondary importance compared with the quality of the material being dried.

NOTATION

G_I, ΔG, initial amount of moisture and loss of moisture in specimen; τ, $\Delta\tau_{phase}$, time and phase time; $\bar{u}_m$, $\bar{u}_s$, mean moisture contents of material and sorbent; l, h, length and thickness of layer, respectively.

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ANALYTIC INVESTIGATION OF HEAT AND MASS TRANSFER UNDER VARIED DRYING CONDITIONS

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UDC 66.047.35

Formulas are obtained for heat and mass flows at the boundary of the semispace during the second drying period with the temperature and mass-transfer potential remaining constant at that boundary.

In a number of engineering processes the basis is provided by heat and mass transfer (drying, conditioning, rectification, etc.). In the present article an analytic investigation is carried out of heat and mass transfer during the process of drying under varied conditions.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 6, pp. 1075-1080, June, 1976. Original article submitted May 26, 1975.

Let us consider a system of differential equations for heat and mass transfer [1]:

$$\frac{\partial\theta_1}{\partial t} = a_1 \frac{\partial^2\theta_1}{\partial x^2} + b_1 \frac{\partial\theta_2}{\partial t}, \quad \frac{\partial\theta_2}{\partial t} = a_2 \frac{\partial^2\theta_2}{\partial x^2} + b_2 \frac{\partial^2\theta_1}{\partial x^2}, \tag{1}$$

$$(a_1 + b_1 b_2 + a_2)^2 \neq 4a_1 a_2$$

in the domain D_1 $(0 < x < \infty, t > 0)$ under the following boundary conditions:

$$\theta_i(x, 0) = \theta_{i0} \quad (0 < x < \infty; \quad i = 1, 2), \tag{2}$$

$$-\lambda_1 \frac{\partial\theta_1(0, t)}{\partial x} = q_1(t), \quad \lambda_2 \frac{\partial\theta_2(0, t)}{\partial x} + \lambda_2\delta \frac{\partial\theta_1(0, t)}{\partial x} = q_2(t) \quad (t > 0), \tag{3}$$

$$\frac{\partial\theta_i(\infty, t)}{\partial x} = \theta_i(\infty, t) = 0 \quad (i = 1, 2), \tag{4}$$

where θ_1 is the temperature; θ_2 is the mass-transfer potential; q_1 is the heat flow; q_2 is the mass flow; x is the space coordinate; t is time; a_1, a_2, b_1, b_2, λ_1, λ_2, δ are well-known constant values [1].

During the time t_1 the drying takes place with a constant heat flow q_{10} and a constant mass flow q_{20} which results in the temperature and the mass-transfer potential reaching the values θ_{11} and θ_{21}, respectively (the first drying period). Rules will be established for modifying the heat and mass flows so that the temperature θ_{11} and the mass-transfer potential θ_{21} are maintained on the surface (the second drying period).

During the first period one has

$$q_1(t) = q_{10}, \quad q_2(t) = q_{20} \tag{5}$$

in the boundary conditions (3). We shall determine the functions $\theta_1(x, t)$ and $\theta_2(x, t)$ that satisfy the system of equations (1), the initial conditions (2), and the boundary conditions (3), (4), and (5). The solution is found by simultaneous application of Fourier and Laplace integral transformations. As regards the variable x, the Fourier cosine transform is used:

$$F\theta(x, t) = \frac{2}{\pi} \int_0^\infty \theta(\xi, t) \cos\omega\xi d\xi. \tag{6}$$

When the image $F\theta(x, t)$ is known the function $\theta(x, t)$ is found by using the formula

$$\theta(x, t) = \int_0^\infty [F\theta(x, t)] \cos\omega x d\omega. \tag{7}$$

For the transform of the second derivative one has

$$F \frac{\partial^2\theta(x, t)}{\partial x^2} = -\omega^2 F\theta(x, t) - \frac{2}{\pi} \frac{\partial\theta(0, t)}{\partial x}. \tag{8}$$

As regards the variable t one uses the Laplace transform

$$L\theta(x, t) = \int_0^\infty \exp(-pt)\,\theta(x, t)\,dt, \tag{9}$$

$$L \frac{\partial\theta(x, t)}{\partial t} = pL\theta(x, t) - \theta(x, 0). \tag{10}$$

By applying the integral transforms (6) and (9) successively together with the formulas (8) and (10), the initial conditions (2) and the boundary conditions (3) and (5), the original problem is reduced to an algebraic system; one then finds with the aid of the inverse Laplace transform

$$F\theta_i(x, t) = \sum_{k,j=1}^{2} \left(A_{kj}^i Fv_{kj} + \frac{2}{\pi} B_{kj}^i Fw_{kj} \right) \quad (i = 1, 2), \tag{11}$$

where

$$Fv_{kj} = \exp(-\alpha_k\omega^2 t)\, F\theta_{j0}, \quad Fw_{kj} = \int_0^t \exp(-\alpha_k\omega^2\tau)\, q_{j0} d\tau,$$

$$\alpha_k = \frac{1}{2}\,[a_1 + b_1 b_2 + a_2 + (-1)^{k+1}\sqrt{(a_1 + b_1 b_2 + a_2)^2 - 4a_1 a_2}\,]$$
$$(k = 1,\ 2).$$

The constant coefficients A^i_{kj}, B^i_{kj} (i, k, j = 1, 2) are given by the formulas

$$A^1_{11} = (\alpha_1 - a_2)\, z,\ A^1_{21} = (a_2 - \alpha_2)\, z,\ A^1_{12} = -A^1_{22} = a_2 b_1 z,$$

$$B^1_{11} = \frac{a_1}{\lambda_1}(\alpha_1 - a_2)\, z,\ B^1_{12} = -\frac{a_2}{\lambda_2} b_1 \alpha_1 z,\ B^1_{21} = \frac{a_1}{\lambda_1}(a_2 - \alpha_2)\, z,$$

$$B^1_{22} = \frac{a_2}{\lambda_2} b_1 \alpha_2 z,$$

$$A^2_{11} = -A^2_{21} = -b_2 z,\ A^2_{12} = (\alpha_1 - a_1 - b_1 b_2)\, z,\ A^2_{22} = (b_1 b_2 + \alpha_2 - a_1)\, z,$$

$$B^2_{11} = -B^2_{21} = -\frac{a_1}{\lambda_1} b_2 z,\ B^2_{12} = \frac{a_2}{\lambda_2}(a_1 - \alpha_1)\, z,\ B^2_{22} = \frac{a_2}{\lambda_2}(\alpha_2 - a_1)\, z,$$

$$z = \frac{1}{\alpha_1 - \alpha_2}.$$

The application of the inversion formula (7) to the expression (11) results in

$$\theta_i(x,\ t) = \sum_{k,j=1}^{2} (A^i_{kj} v_{kj} + B^i_{kj} w_{kj}) \quad (i = 1,\ 2), \tag{12}$$

where

$$v_{kj} = \frac{1}{2}\int_0^\infty [\chi(x - \xi,\ \alpha_k t) + \chi(x + \xi,\ \alpha_k t)]\,\theta_{j0} d\xi = \theta_{j0}, \tag{13}$$

$$w_{kj} = \int_0^\infty \chi[x,\ \alpha_k(t - \tau)]\, q_{j0} d\tau = \frac{q_{j0}}{\alpha_k}\left[\frac{2\sqrt{\alpha_k t}}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4\alpha_k t}\right) - x \operatorname{erfc}\frac{x}{2\sqrt{\alpha_k t}}\right], \tag{14}$$

$$\chi(x,\ y) = \frac{1}{\sqrt{\pi y}} \exp\left(-\frac{x^2}{4y}\right). \tag{15}$$

It follows from the formula (12) that on the boundary (for x = 0) the temperature and the mass-transfer potential are given by

$$\theta_i(0,\ t) = \theta_{i0} + \sum_{k,j=1}^{2} 2B^i_{kj} \frac{q_{j0}\sqrt{t}}{\sqrt{\alpha_k \pi}} \quad (i = 1,\ 2). \tag{16}$$

From (16) one can find the time during which the heating should be maintained so that on the surface the temperature θ_{11} is obtained:

$$t_1 = \frac{(\theta_{11} - \theta_{10})^2 \pi}{4\left(\sum_{k,j=1}^{2} B^1_{kj} \frac{q_{j0}}{\sqrt{\alpha_k}}\right)^2}; \tag{17}$$

and also the time required to establish the mass-transfer potential θ_{21} on the surface:

$$t_2 = \frac{(\theta_{21} - \theta_{20})^2 \pi}{4\left(\sum_{k,j=1}^{2} B^2_{kj} \frac{q_{j0}}{\sqrt{\alpha_k}}\right)^2}. \tag{18}$$

The equality $t_1 = t_2$ yields

$$q_{20} = q_{10} \frac{\frac{a_1}{\lambda_1}\left[\Delta\theta_2\left(\frac{\alpha_1 - a_2}{\sqrt{\alpha_1}} + \frac{a_2 - \alpha_2}{\sqrt{\alpha_2}}\right) - \Delta\theta_1 b_2 \frac{\sqrt{\alpha_2} - \sqrt{\alpha_1}}{\sqrt{\alpha_1 \alpha_2}}\right]}{\frac{a_2}{\lambda_2}\left[\Delta\theta_1\left(\frac{a_1 - \alpha_1}{\sqrt{\alpha_1}} + \frac{\alpha_2 - a_1}{\sqrt{\alpha_2}}\right) - \Delta\theta_2 b_1 (\sqrt{\alpha_2} - \sqrt{\alpha_1})\right]}, \tag{19}$$

where $\Delta \theta_i = \theta_{i1} - \theta_{i0}$ (i = 1, 2).

Let us now consider the system (1) in the domain D_2 ($0 < x < \infty$, $t > t_1$) under the boundary conditions (20), (21), and (4) (the second drying period):

$$\theta_i(x, t_1) = \theta_{i0} + \sum_{k,j=1}^{2} B_{kj}^i \frac{q_{j0}}{\alpha_k} \left[\frac{2\sqrt{\alpha_k t}}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4\alpha_k t_1} \right) - \right.$$

$$\left. - x \operatorname{erfc} \frac{x}{2\sqrt{\alpha_k t_1}} \right] = \psi_i(x) \quad (0 < x < \infty;\ i = 1, 2), \tag{20}$$

$$\theta_i(0, t) = \theta_{i0} + \sum_{k,j=1}^{2} B_{kj}^i \frac{q_{i0} 2\sqrt{t_1}}{\sqrt{\alpha_k \pi}} = \theta_{i1} \quad (t > t_1). \tag{21}$$

One now determines the functions $\theta_1(x, t)$ and $\theta_2(x, t)$ which satisfy Eqs. (1), the initial conditions (20), and the boundary conditions (21) and (4).

As regards the variable x, the Fourier sine transform is applied:

$$F\theta(x, t) = \frac{2}{\pi} \int_0^\infty \theta(\xi, t) \sin \omega\xi d\xi. \tag{22}$$

The function $\theta(x, t)$, if its image $F\theta(x, t)$ is known, can be determined with the aid of the formula

$$\theta(x, t) = \int_0^\infty [F\theta(x, t)] \sin \omega x d\omega. \tag{23}$$

One has for the transform of the second derivative

$$F \frac{\partial^2 \theta}{\partial x^2} = -\omega^2 F\theta + \frac{2\omega}{\pi} \theta(0, t). \tag{24}$$

As regards the variable t, the Laplace transform (9) and (10) is used.

By a successive application of the transform (22) and (9) together with (23), (10), (20), (21), and (4), the original problem is reduced to an algebraic system whose solution as a result of using the inverse Laplace transform is given by

$$F\theta_i(x, t) = \sum_{k,j=1}^{2} \left(C_{kj}^i FP_{kj} + \frac{2}{\pi} D_{kj}^i FQ_{kj} \right) \quad (i = 1, 2), \tag{25}$$

where

$$FP_{kj} = \exp(-\alpha_k \omega^2 t) F\psi_j(x), \quad FQ_{kj} = \int_0^t \exp(-\alpha_k \omega^2 \tau) \theta_{j1} d\tau.$$

The constant coefficients C_{kj}^i, D_{kj}^i (i, k, j = 1, 2) are given by the formulas

$$C_{11}^1 = (\alpha_1 - a_2) z, \quad C_{21}^1 = (a_2 - \alpha_2) z, \quad C_{12}^1 = -C_{22}^1 = -b_1 a_2 z,$$

$$D_{11}^1 = [\alpha_1 (b_1 b_2 + a_1) - a_1 a_2] z, \quad D_{12}^1 = -b_1 a_2 \alpha_1 z, \quad D_{21}^1 = [a_1 a_2 -$$

$$- \alpha_2 (b_1 b_2 + a_1)] z, \quad D_{22}^1 = b_1 a_2 \alpha_2 z,$$

$$C_{11}^2 = -C_{21}^2 = b_2 z, \quad C_{12}^2 = (\alpha_1 - a_1 - b_1 b_2) z, \quad C_{22}^2 = (a_1 + b_1 b_2 - \alpha_2) z,$$

$$D_{11}^2 = b_2 \alpha_1 z, \quad D_{21}^2 = -b_2 \alpha_2 z, \quad D_{12}^2 = a_2 (\alpha_1 - a_1) z, \quad D_{22}^2 = a_2 (a_1 - \alpha_2) z,$$

$$z = \frac{1}{\alpha_1 - \alpha_2}.$$

Applying the inversion formula (23) to the expression (25), one obtains

$$\theta_i(x, t) = \sum_{k,j=1}^{2} (C_{kj}^i P_{kj} + D_{kj}^i Q_{kj}) \quad (i = 1, 2), \tag{26}$$

where

$$P_{kj}=\frac{1}{2}\int_{0}^{\infty}[\chi(x-\xi,\ \alpha_k t)+\chi(x+\xi,\ \alpha_k t)]\,\psi_j(\xi)\,d\xi, \tag{27}$$

$$Q_{kj}=\frac{x}{2\sqrt{\pi\alpha_k^3 t^3}}\exp\left(-\frac{x^2}{4\alpha_k t}\right)\overset{t}{*}\theta_{j1}, \tag{28}$$

where the symbol $\overset{t}{*}$ denotes the convolution:

$$f(t)\overset{t}{*}g(t)=\int_{0}^{t}f(t-\tau)\,g(\tau)\,d\tau. \tag{29}$$

One now proceeds to determine the unknown functions $q_1(t)$ and $q_2(t)$ from the boundary conditions (3).

By differentiating (23) with respect to x one finds

$$\left.\frac{\partial\theta_i(x,\ t)}{\partial x}\right|_{x=0}=\sum_{k,j=1}^{2}(C_{kj}^i R_{kj}+D_{kj}^i S_{kj})\quad(i=1,\ 2), \tag{30}$$

where

$$R_{kj}=\frac{1}{2}\int_{0}^{\infty}\frac{\partial}{\partial x}[\chi(x-\xi,\ \alpha_k t)+\chi(x+\xi,\ \alpha_k t)]\,\psi_j(\xi)\,d\xi\Big|_{x=0}, \tag{31}$$

$$S_{kj}=\frac{1}{2\sqrt{\pi\alpha_k^3 t^3}}\frac{\partial}{\partial x}\left[x\exp\left(-\frac{x^2}{4\alpha_k t}\right)\overset{t}{*}\theta_{j1}\right]\Big|_{x=0}. \tag{32}$$

The relation (31) is transformed by integrating by parts. Taking into account that

$$\frac{\partial}{\partial x}\exp\left[-\frac{(\xi-x)^2}{4\alpha t}\right]=-\frac{\partial}{\partial\xi}\exp\left[-\frac{(\xi-x)^2}{4\alpha t}\right],$$

$$\frac{\partial}{\partial x}\exp\left[-\frac{(\xi+x)^2}{4\alpha t}\right]=\frac{\partial}{\partial\xi}\exp\left[-\frac{(\xi+x)^2}{4\alpha t}\right],$$

one obtains from the formula (20)

$$\frac{\partial\psi_i(\xi)}{\partial\xi}=-\sum_{k,j=1}^{2}B_{kj}^i\frac{q_{j0}}{\alpha_k}\operatorname{erfc}\left(\frac{\xi}{2\sqrt{\alpha_k t}}\right),\quad\psi_i(0)=\theta_{i1},$$

and then

$$R_{kj}=\frac{\theta_{j1}}{\sqrt{\pi\alpha_k t}}+\sum_{n,m=1}^{2}B_{nm}^j\frac{q_{j0}}{\alpha_k}\left[1-\frac{2}{\pi}\operatorname{arctg}\sqrt{\frac{t}{t_1}}\right]. \tag{33}$$

By transforming (32) one finds

$$S_{kj}=\frac{\theta_{j1}}{\alpha_k\sqrt{\pi\alpha_k t}}. \tag{34}$$

Finally, using (3), (30), (33), and (34), and bearing in mind the expressions for the constant coefficients C_{kj}^i, D_{kj}^i, B_{nm}^j, one obtains

$$q_i(t)=\nu\lambda_i\frac{\theta_{i1}}{\sqrt{t}}+\frac{a_1 a_2}{\alpha_1\alpha_2}q_{i0}\left[1-\frac{2}{\pi}\operatorname{arctg}\sqrt{\frac{t}{t_1}}\right], \tag{35}$$

where

$$\nu=\left[(a_1+a_2+b_1 b_2)(\sqrt{\alpha_2}-\sqrt{\alpha_1})+a_1 a_2\left(\frac{\sqrt{\alpha_1}}{\alpha_2}-\frac{\sqrt{\alpha_2}}{\alpha_1}\right)+\alpha_2\sqrt{\alpha_1}-\alpha_1\sqrt{\alpha_2}\right]\left[(\alpha_1-\alpha_2)\sqrt{\alpha_1\alpha_2\pi}\right]^{-1};$$

in particular, for $b_1 = b_2 = 0$ ($\alpha_1 = a_1$, $\alpha_2 = a_2$, $\nu = 0$) it follows from (35) that

$$q_i(t) = q_{i0}\left[1 - \frac{2}{\pi}\,\mathrm{arctg}\sqrt{\frac{t}{t_1}}\right] \quad (i = 1,\ 2). \tag{36}$$

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TEMPERATURE MEASUREMENT USING THERMISTOR WITH PULSED OPERATION OF CIRCUIT CONTAINING THERMISTOR AND LINEAR RESISTOR

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UDC 536.63 : 621.316.825 : 621.3.011.1

A method is considered for determining the basic parameters characterizing a pulsed thermistor–linear resistor temperature-measuring circuit and ensuring increased sensitivity to temperature changes while conserving a given accuracy of measurement.

The main demands imposed on the design of temperature-measuring apparatuses reduce to sensitivity and accuracy. In the event that a semiconductor thermistor is used as the temperature sensor, it turns out that these demands are contradictory, since a high sensitivity of the apparatus requires a significant current flow in the sensor circuit; this current heats up the thermistor and so gives rise to a systematic measurement error. This error is usually reduced at the expense of the sensitivity, by reducing the current, which for microthermistors varies from one to a few tens of microamps.

The dilemma can be obviated to a large degree by pulse operation of the thermistor-containing measuring circuit. If the supply of the $R_T - R$ circuit (i.e., the thermistor–linear resistor circuit) is pulsed in such a manner that the mean power supplied equals the power supplied at dc, then the amplitude of the pulses of supply current or voltage may be increased over the dc value by a factor of $1/\sqrt{\gamma}$ (where γ is the duty factor, the ratio of pulse duration to the pulse repetition period). The heating of the thermistor that occurs in this case too by the current passing through it can be estimated from the curve of the transient process.

The theory of pulse systems is well developed and is presented in detail in Tsypkin's books [1, 2], for example.

Transient processes in thermistor circuits for pulse-type variations of the input quantities are considered in [3-6].

Nonetheless, the practical realization of the pulse method of temperature measurement using semiconductor thermistors has been frustrated until recently due to the absence of a simple and reliable high-speed

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 6, pp. 1081-1088, June, 1976. Original article submitted June 16, 1975.